Marginal rate of substitution

Let the production function be $q = 6a^{2/3}b^{1/3}$ where q is the quantity produced when employing quantities a and b of two inputs A and B. If generally 64 units of A and 8 units of B are used, what approximate variation should occur in the quantity of input B if the quantity of input A is increased by 1 unit so that the product remains constant, i.e., $\Delta q = 0$?

To answer the question, we have to calculate the marginal rate of substitution, which is composed of derivatives. Therefore, we calculate the partial derivatives.

$$\frac{\partial q}{\partial a} = \frac{2}{3}6a^{-1/3}b^{1/3}$$

$$\frac{\partial q}{\partial b} = \frac{1}{3}6a^{2/3}b^{-2/3}$$

Now we calculate the marginal rate of substitution. As we are talking about increasing the quantity of input A, the MRS has the following formula:

$$MRS = |-\frac{\partial q/\partial a}{\partial q/\partial b}| = \frac{\frac{2}{3}6a^{-1/3}b^{1/3}}{\frac{1}{3}6a^{2/3}b^{-2/3}} = \frac{(2/3)b}{(1/3)a} = \frac{2b}{a}$$

Evaluated at the point:

$$\frac{28}{64} = 0.25$$

Therefore, if the quantity of input A is increased by one unit, input B must decrease by 0.25 to keep production constant.